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State estimation for power systems embedded with FACTS devices and MTDC systems by a sequential solution approach

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Abstract

This paper reports the development of a novel and effective approach in state estimation for power systems with flexible AC transmission system (FACTS) and multi-terminal DC (MTDC) systems, called improved sequential method. The proposed approach is sequential in nature in which the FACTS and MTDC systems without neglecting the coupling submatrices in the gain matrix, it exhibits good convergence characteristics compared to conventional techniques. The variables and measurement equations of the FACTS and MTDC systems related to the problem formulation are discussed. The effectiveness of the proposed algorithm is demonstrated in this paper with extensive testing in several test systems and the results are compared with the other state estimators.

Keywords: MTDC; FACTS; State estimation; EMS

1. Introduction

In recent years, environmental considerations, growing energy demand and the recent trend in deregulation of electric utility require more effective use of transmission system. New technology that can improve the capability of power delivery and efficiently control power flows across specified lines has become necessary. In this context, flexible AC transmission system (FACTS) technology has achieved very rapid advances to meet such objectives, and many FACTS devices, such as thyristor controlled series compensation (TCSC), thyristor controlled phase shifting transformer (TCPST), and unified power flow controller (UPFC), have been developed. High-voltage DC (HVDC) transmission systems, whose advantages are in bulk power transmission over long distances and connection of systems with different frequencies, have already been in use before FACTS was introduced, and have since then become mature technology.

State estimation plays an important role in energy management system (EMS), which forms the basis of analyzing network security. Numerous algorithms have been published on the state estimation applied to power systems [1,2]. However, very limited efforts have been made to study the impacts of FACTS devices and systems [1,2]. However, very limited efforts have been made to study the impacts of FACTS devices and HVDC links on power system state estimation. Because of the significance of state estimation, it is imperative that an accurate representation of the power system model including FACTS devices and HVDC systems be developed. The main issue in this study is thus to discuss state estimation for power systems embedded with FACTS devices and multi-terminal DC (MTDC) systems.

With the introduction FACTS devices and MTDC systems, new variables are appended to the solution state vector X of state estimation. Furthermore, compared with the conventional AC power system measurements, there are additional measurements associated with these FACTS devices and MTDC systems. Several papers [3–5] on AC/DC integrated power system state estimation provide some clues to update the existing AC state estimation for considering the impacts of FACTS devices and MTDC systems. Jagatheesan and Duraiswamy [3] proposed a sequential state estimation

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method for the AC/DC system, in which the AC system was solved by traditional fast decoupled state estimators (FDSE), and followed by DC system state estimation. The method is simple and can be easily appended to existing AC fast decoupled state estimators. However, the method decouples AC and DC systems by neglecting the coupling blocks in the gain matrix and thus would degrade the convergence characteristics. Roy et al. [4] presented an algorithm to solve the AC and DC systems simultaneously, which used the weighted least square (WLS) technique. The method is robust and reliable, but maybe it increases the complexity of program and is hard to apply existing AC state estimation software. Sinha et al. [5] proposed to sue the rectangular coordinates and decoupled the WLS gain matrix by shifting the coupled blocks in the right-hand side and then solved the AC and DC systems sequentially. In state estimation incorporated FACTS devices and MTDC systems, it is advantageous to propose a method, which has good convergence characteristics, and take advantage of the accumulated experience in AC power system state estimation as well as reduce software development efforts.

Similar to the exact decoupling approach, presented by Monticelli and Garcia [6] in which the coupling matrices between real power and reactive power in the gain matrix are not directly made zero when the real power and reactive power parts are decoupled, a new method, called improved sequential method, is presented in this paper for state estimation with FACTS devices and MTDC systems. The proposed method decouples the AC, FACTS and MTDC parts by modifying the full gain matrix into several block diagonal matrices through a mathematical process. The main features of the proposed technique are:

- After partitioning the AC part, FACTS part and MTDC part in the gain matrix, a sequential iteration form can be used: in other words, the AC system can be solved followed by the FACTS devices and MTDC systems. We can thus incorporate the existing state estimation programs to include FACTS devices and MTDC systems, and reduce software development efforts and maintenance costs.
- The method is a general one due to its being capable of extending to include more new emerging FACTS control devices.
- The new method is developed from the WLS gain matrix, so it will maintain the good convergence property of the conventional WLS method.

This paper is organized as follows. The review of state estimation is given in Section 2. The problem formulation and the proposed method of state estimation for the power systems embedded with FACTS devices and MTDC systems are given in Section 3. The development of the mathematical models of MTDC systems and FACTS devices are given in Section 4 and Section 5, respectively. The digital simulation results are presented in Section 6, and the conclusions of this paper are discussed in Section 7.

2. Review of state estimation algorithms

Static state estimation is an algorithm for converting redundant noisy, uncertain measurements into a reliable estimate of the state of a power system. The mathematical model of state estimation is based on the mathematical relations between the measurements and the state variables. The set of telemetered measurements Z is related to its n-dimensional state vector X as follows:

$$Z = h(X) + \eta \tag{1}$$

where h represents a m-dimensional nonlinear vector function and η denotes measurements error vector with zero mean and covariance matrix R, which is a diagonal matrix with diagonal elements $r_{ii} = \sigma_{ii}^2$, where σ_{ii}^2 is the variance of the ith measurement. Eq. (1) involves electric quantities from AC system, FACTS devices and MTDC systems. The estimate $\hat{X}$ is obtained using WLS criteria by minimizing the objective function

$$J(X) = [Z - h(X)]^T R^{-1} [Z - h(x)] \tag{2}$$

According to the basic WLS method, the solution $\hat{X}$ is obtained by an iterative procedure as follows:

$$G(X^k)\Delta X^k = H^T(X^k) R^{-1} [Z - h(X^k)] \tag{3}$$

$$X^{k+1} = X^k + \Delta X^k \tag{4}$$

where H is the Jacobian matrix of the measurement functions, and gain matrix $G(X^k)$ is given by

$$G\,(X^k) = H^T(X^k)\, R^{-1} H\,(X^k) \tag{5}$$

From [6], it is noted that Eq. (3) is the solution of the linear WLS problem (the superscript k is dropped for convenience)

$$\min_{\Delta X} J(\Delta X) = [\Delta\bar{Z} - \bar{H}\Delta X]^T [\Delta\bar{Z} - \bar{H}\Delta X] \tag{6}$$

where $\bar{H} = R^{-1/2}H$ and $\Delta\bar{Z} = R^{-1/2}H\Delta Z = R^{-1/2}$ $[Z - H(x)]$. The least square problem is a desired (minimum norm) solution to the over-determined system of linear equations,

$$\bar{H}\Delta X \cong \Delta\bar{Z} \tag{7}$$

Then, by using the notion of the pseudo-inverse $\bar{H}^+ = (\bar{H}^T\bar{H})^{-1}H^T$ of $\bar{H}$, Eq. (3) may be expressed as the solution of Eq. (7):

$$\Delta X = \bar{H}^+ \Delta\bar{Z} \tag{8}$$

3. Problem formulation

3.1. Proposed method

The state variables of state estimation including FACTS and MtDC systems consist of three sets: AC state variables X_{ac} (bus voltage magnitudes and phase angles), FACTS state variables X_F, and DC state variables X_{dc} $\Delta X = [\Delta X_{ac}, \Delta X_F, \Delta X_{dc}]^T$ can be expressed as

$$\begin{bmatrix} \bar{H}_{ac} & \bar{H}_{ac-F} & \bar{H}_{ac-dc} \\ \bar{H}_{F-ac} & \bar{H}_F & 0 \\ \bar{H}_{dc-ac} & 0 & \bar{H}_{dc} \end{bmatrix} \begin{bmatrix} \Delta X_{ac} \\ \Delta X_F \\ \Delta X_{dc} \end{bmatrix} \cong \begin{bmatrix} \Delta \bar{Z}_{ac} \\ \Delta \bar{Z}_F \\ \Delta \bar{Z}_{dc} \end{bmatrix} \tag{9}$$

Same as the approach to decouple real power and reactive power parts in the AC state estimation of [6], this new approach does not zero coupling Jacobian submatrices $\bar{H}_{ac-dc}$, $\bar{H}_{dc-ac}$, $\bar{H}_{ac-F}$ and $\bar{H}_{F-ac}$. Instead, after taking coupling effects into account precisely, a sequential procedure to solve Eq. (9) can be adopted, in which corrections ΔX_{ac}, ΔX_F and ΔX_{dc}, are computed separately, without any other major approximation.

By the Gaussian elimination process, the sub-blocks and of Jacobian matrix in Eq. (9) are eliminated, respectively. Further, we can derive the following decoupled form after shifting the coupling blocks $\bar{H}_{F-ac}$, and $\bar{H}_{dc-ac}$ to the right-hand side.

$$\begin{bmatrix} \bar{H}_{ac} & 0 & 0 \\ 0 & \bar{H}_F & 0 \\ 0 & 0 & \bar{H}_{dc} \end{bmatrix} \begin{bmatrix} \Delta X_{ac} \\ \Delta X_F \\ \Delta X_{dc} \end{bmatrix} \cong \begin{bmatrix} \Delta \tilde{Z}_{ac} \\ \Delta \tilde{Z}_F \\ \Delta \tilde{Z}_{dc} \end{bmatrix} \tag{10}$$

where

$\tilde{H}_{ac} = \bar{H}_{ac-dc} - \bar{H}_{ac-dc}\bar{H}^+_{dc}\bar{H}_{dc-ac} - \bar{H}_{ac-F}\bar{H}^+_F$

$\bar{H}_{F-ac}$, $\Delta\tilde{Z}_{ac} - \tilde{H}_{ac-dc}\bar{H}^+_{dc}\,\Delta\bar{Z}_{dc} - \bar{H}_{ac-F}\bar{H}_{ac-F}\bar{H}^+_F\Delta\bar{Z}_F$,

$\bar{H}^+_{dc} = [\bar{H}^T_{dc}\bar{H}_{dc}]^{-1}\bar{H}^T_{dc}$, $\bar{H}^+_F = [\bar{H}^T_F\bar{H}_F]^{-1}\bar{H}^T_F$, $\Delta\tilde{Z}_{dc}$

$= \Delta\bar{Z}_{dc} - \bar{H}_{dc-ac}\Delta X_{ac}$, $\Delta\tilde{Z}_F = \Delta\bar{Z}_F - \bar{H}_{F-ac}\Delta X_{ac}$.

Eq. (10) is equivalent to Eq. (9) provided that the measurements present no errors, and can be solved by the following three-step sequential algorithm.

3.1.1. Step 1

$$\begin{cases} \Delta X^{\text{temp}\,(k)}_{dc} = \bar{H}^{+\,(k)}_{dc}\,\Delta\bar{Z}_{dc}(X^{(k)}_{ac}, X^{(k)}_{dc}) \\ X^{\text{temp}\,(k+1)}_{dc} = X^{(k)}_{dc} + \Delta X^{\text{temp}\,(k)}_{dc} \end{cases} \tag{11}$$

$$\begin{cases} \Delta X^{\text{temp}\,(k)}_{F} = \bar{H}^{+\,(k)}_{F}\,\Delta\bar{Z}_{F}(X^{(k)}_{ac}, X^{(k)}_{F}) \\ X^{\text{temp}\,(k+1)}_{F} = X^{(k)}_{F} + \Delta X^{\text{temp}\,(k)}_{F} \end{cases} \tag{12}$$

3.1.2. Step 2

In the Appendix (A-1) it is shown that

$$\Delta\tilde{Z}_{ac}(X^{(k)}_{ac}, X^{(k)}_{dc}, X^{(k)}_F) = \Delta\bar{Z}_{ac}(X^{(k)}_{ac}, X^{\text{temp}\,(k+1)}_{dc}, X^{\text{temp}\,(k+1)}_F) \tag{13}$$

So

$$\begin{cases} \Delta X^{(k)}_{ac} = \tilde{H}^{+\,(k)}_{ac}\,\Delta\tilde{Z}^{(k)}_{ac} = \tilde{H}^{+\,(k)}_{ac}\,\Delta\bar{Z}_{ac}(X^{(k)}_{ac}, X^{\text{temp}\,(k+1)}_{dc}, X^{\text{temp}\,(k+1)}_F) \\ X^{(k+1)}_{ac} = X^{(k)}_{ac} + \Delta X^{(k)}_{ac} \end{cases} \tag{14}$$

3.1.3. Step 3

$$\begin{cases} \Delta X^{\text{com}\,(k)}_{dc} = -\bar{H}^{+\,(k)}_{dc}\,\bar{H}^{(k)}_{dc-ac}\,\Delta X^{(k)}_{dc} \\ X^{(k+1)}_{dc} = X^{\text{temp}\,(k+1)}_{dc} + \Delta X^{\text{com}\,(k)}_{dc} \end{cases} \tag{15}$$

$$\begin{cases} \Delta X^{\text{com}\,(k)}_{F} = -\bar{H}^{+\,(k)}_{F}\,\bar{H}^{(k)}_{F-ac}\,\Delta X^{(k)}_{F} \\ X^{(k+1)}_{F} = X^{\text{temp}\,(k+1)}_{F} + \Delta X^{\text{com}\,(k)}_{F} \end{cases} \tag{16}$$

Next we study the step 1 of the $(k+1)$th iteration.

3.1.4. Step 1

$$\begin{cases} \Delta X^{\text{temp}\,(k)}_{dc} = -\bar{H}^{+\,(k+1)}_{dc}\,\Delta\bar{Z}_{dc}(X^{(k+1)}_{ac}, X^{(k+1)}_{dc}) \\ X^{\text{temp}\,(k+2)}_{dc} = X^{(k+1)}_{dc} + \Delta X^{\text{temp}\,(k+1)}_{dc} \end{cases} \tag{17}$$

Here it is reasonable to assume that Jacobian matrix H_{dc} is appoximately unchanged in that the values of DC state variables X_{dc} are little changed between step 3 of kth iteration and step 1 of $(k+1)$th iteration. By adding $\Delta^{\text{temp}\,(k+1)}_{dc} + \Delta X^{\text{com}\,(k)}_{dc}$, we can drive

$$\Delta X^{\text{com}\,(k)}_{dc} + \Delta X^{\text{temp}\,(k+1)}_{dc} = \bar{H}^+_{dc}\,[\Delta\bar{Z}_{dc}\,(X^{(k+1)}_{ac}, X^{(k+1)}_{dc}) - \bar{H}_{dc}\Delta X^{\text{com}\,(k)}_{dc} - \bar{H}_{dc-ac}\Delta X^{(k)}_{ac}] = \bar{H}^+_{dc}\,[\Delta\bar{Z}_{dc}\,(X^{(k+1)}_{ac}, X^{\text{temp}(k+1)}_{dc}) - \bar{H}_{dc}\Delta X^{\text{com}\,(k)}_{dc} - \bar{H}_{dc-ac}\Delta X^{(k)}_{ac}] \tag{18}$$

In the Appendix it is shown that

$$\bar{H}^+_{dc}(\bar{H}_{dc}\Delta X^{\text{com}\,(k)}_{dc} + \bar{H}_{dc-ac}\Delta X^{(k)}_{ac}) = 0 \tag{19}$$

From Eq. (18) and Eq. (19) we finally obtain

$$\Delta X^{\text{com}\,(k)}_{dc} + \Delta X^{\text{temp}\,(k+1)}_{dc} \approx \bar{H}^+_{dc}\Delta\bar{Z}_{dc}\,(X^{(k+1)}_{ac}, X^{\text{temp}\,(k+1)}_{dc}) \tag{20}$$

Similarly,

$$\Delta X^{\text{com}\,(k)}_{F} + \Delta X^{\text{temp}\,(k+1)}_{F} \approx \bar{H}^+_{F}\,\Delta\bar{Z}_{F}\,(X^{(k+1)}_{ac}, X^{\text{temp}\,(k+1)}_{F}) \tag{21}$$

Therefore, the computations performed as Step 3 are automatically taken into account in the following iteration, Then the 3-step iteration form can be altered as follows.

3.1.5. Step 1

$$\begin{cases}\Delta X_{\mathrm{dc}}^{(k)} = \bar{H}_{\mathrm{dc}}^{+\,(k)}\,\Delta\bar{Z}_{\mathrm{dc}}^{(k)}(X_{\mathrm{ac}}^{(\mathrm{k})},X_{\mathrm{dc}}^{(\mathrm{k})})\\ \quad X_{\mathrm{dc}}^{(k+1)} = X_{\mathrm{dc}}^{(k)} + \Delta X_{\mathrm{dc}}^{(k)}\end{cases} \tag{22}$$

$$\begin{cases}\Delta X_{\mathrm{F}}^{(k)} = \bar{H}_{\mathrm{dc}}^{+\,(k)}\,\Delta\bar{Z}_{\mathrm{F}}\,(X_{\mathrm{ac}}^{(k)},\, X_{\mathrm{F}}^{(k)})\\ \quad X_{\mathrm{F}}^{(k+1)} = X_{\mathrm{F}}^{(k)} + \Delta X_{\mathrm{F}}^{(k)}\end{cases} \tag{23}$$

3.1.6. Step 2

$$\begin{cases}\Delta X_{\mathrm{ac}}^{(k)} = \tilde{H}_{\mathrm{ac}}^{+\,(k)}\,\Delta\tilde{Z}_{\mathrm{ac}}^{(k)} = \tilde{H}_{ac}^{+(k)}\Delta\bar{Z}_{\mathrm{ac}}(X_{\mathrm{ac}}^{(k)},\, X_{\mathrm{dc}}^{(k+1)},\, X_{\mathrm{F}}^{(k+1)})\\ \qquad X_{\mathrm{ac}}^{(k+1)} = X_{\mathrm{ac}}^{(k)} + \Delta X_{\mathrm{ac}}^{(k)}\end{cases} \tag{24}$$

Eqs. (22)–(24) are solved alternately and denote the improved sequential method proposed to handle power systems embedded with FACTS devices and MTDC systems. From the above derivation procedure, we can see that the improved sequential iteration form of Step 1 and Step 2 is obtained from WLS iteration form of Eq. (9). The coupling matrices $\bar{H}_{\mathrm{ac-dc}}$ and $\bar{H}_{\mathrm{dc-ac}}$, and $\bar{H}_{\mathrm{F-ac}}$ are not ignored like the FDSE method. Instead, that coupling is correctly considered during the decoupled computations of the corrections ΔX_{ac}, ΔX_{dc} and ΔX_{F}. Moreover, the simple sequential iteration scheme can be adopted while the good convergence chacteristics can be remained. This treatment enables us to utilize exising AC state estimation programs easily.

3.2. Solution steps

According to the improved sequential method described above, equations Eqs. (22)–(24) are solved alternately using the Gaussian forward elimination and backward substitution procedure until the maximum value $|\Delta X|$ is less than the specified tolerance. In the solution scheme, the AC Jacobian matrix $\bar{H}_{\mathrm{ac}}$ should be modified to obtain $\tilde{H}_{\mathrm{ac}}$ in order to consider the DC system and FACTS devices. The detailed solution steps of the proposed algorithm are presented below.

Step 1: Read system data and measurements

Step 2: Initialize the state vector $X_{\mathrm{ac}}^{(0)}$ and $X_{\mathrm{F}}^{(0)}$.

Step 3: Set iteration count $k = 0$ and $[\Delta \mathrm{X}_{\mathrm{ac}}] = [0]$, $[\Delta X_{\mathrm{F}}] = [0]$.

Step 4: Compute MTDC system measurement residual vector $\Delta\bar{Z}_{\mathrm{dc}}^{(k)} = R^{-1/2}\,[Z_{\mathrm{dc}} - h_{\mathrm{dc}}(X_{\mathrm{ac}}^{(k)},\, X_{\mathrm{dc}}^{(k)})]$, and FACTS measurment residual vector $\Delta\bar{Z}_{\mathrm{F}}^{(k)} = R^{-1/2}[Z_{\mathrm{F}} - h_{\mathrm{F}}(X_{\mathrm{ac}}^{(k)},\, X_{\mathrm{F}}^{(k)})]$.

Step 5: Calculate MTDC pseudo-inverse $\bar{H}_{\mathrm{dc}}^{+\,(k)}$ and FACTS Jacobian matrix $\bar{H}_{\mathrm{F}}^{(k)}$

Step 6: Compute MTDC pseudo-inverse $\bar{H}_{\mathrm{dc}}^{+\,(k)}$ and FACTS pseudo-inverse $\bar{H}_{\mathrm{F}}^{(k)}$.

Step 7: Obtain $\Delta X_{\mathrm{dc}}^{(k+1)}$ and $X_{\mathrm{dc}}^{(k+1)} = X_{\mathrm{dc}}^{(k)} + \Delta X_{\mathrm{dc}}^{(k+1)}$, $\Delta X_{\mathrm{F}}^{(k+1)}$ and $X_{\mathrm{F}}^{(k+1)} = X_{\mathrm{F}}^{(k)} + \Delta X_{\mathrm{F}}^{(k+1)}$.

Step 8: Compute AC measurement residual vector $\Delta\tilde{Z}_{\mathrm{ac}}^{(k)} = R^{-1/2}\,[Z_{\mathrm{ac}} - h_{\mathrm{ac}}(X_{\mathrm{ac}}^{(k)},\, X_{\mathrm{dc}}^{(k+1)},\, X_{\mathrm{F}}^{(k+1)})]$.

Step 9: Compute AC system Jacobian matrix $\bar{H}_{\mathrm{ac}}^{(k)}$.

Step 10: Modify some elements of $\bar{H}_{\mathrm{ac}}^{(k)}$ to obtain $\tilde{H}_{\mathrm{ac}}^{(k)}$.

Step 11: Compute AC system gain matrix $G_{\mathrm{ac}}^{(k)} = \tilde{H}_{\mathrm{ac}}^{T(k)}\,\tilde{H}_{\mathrm{ac}}^{(k)}$, triangularize and store factors of $G_{\mathrm{ac}}^{(k)}$.

Step 12: Perform Gaussian forward and backward operations on $\tilde{H}_{\mathrm{ac}}^{T(k)}\,\Delta\tilde{Z}_{\mathrm{ac}}^{(k)}$ to get $\Delta X_{\mathrm{ac}}^{(k+1)}$.

Step 14: Check for convergence. If $|\Delta X_{\mathrm{ac}}^{(k+1)}| \le \varepsilon$, $|\Delta X_{\mathrm{dc}}^{(k+1)}| \le \varepsilon$ and $|\Delta X_{\mathrm{F}}^{(k+1)}| \le \varepsilon$, go to Step 15: otherwise, set $k = k+1$ and go to Step 4.

Step 15: Set $k = k+1$ and print results.

Obviously, it is noted that Step 8 to Step 12 are the same as the traditional AC WLS state estimation method except for the need to alter Jacobian matrix $\bar{H}_{\mathrm{ac}}^{(k)}$.

4. MTDC system analysis

4.1. Measurements

The measurement vector Z of AC/MTDC power system consists of the following telmetered measurements:

- AC system measurments (which are same with the traditional AC state estimation, including real and reactive power injections, real and reactive power flows in lines of AC system, voltage magnitudes and so on).
- DC system measurements.
  The DC system measurement set includes the following: DC current injection I_{di}^{m} at bus i_d, to j_d, DC current flow I_{dij}^{m} from bus i_d to j_d and so on.
- AC/DC interface system measurements.

The following interface measurements are considered: AC current I_{li}^{m} into the converter, off-nominal converter transformer tap ratio a_j^{m}, firing angle θ_i^{m} and so on.

In addition, if needed the AC and DC pseudo-measurments are also used.

4.2. AC/MTDC system variables

The AC state vector, consisting of the complex nodal voltage is $X_{\mathrm{ac}} = [\theta,\ V]^T$, where θ represents the phase angle in all AC system buses except the reference bus phase angle assumed to be zero, V represents the voltage magnitudes. The interface and the DC system state vector is $X_{\mathrm{dc}i}[V_{di}, \mathrm{I}_{di}, a_i, \cos\,\theta_i,\ I_{li},\ \phi_i]^T$ for each converter.

The various quantities of DC system that are involved are:

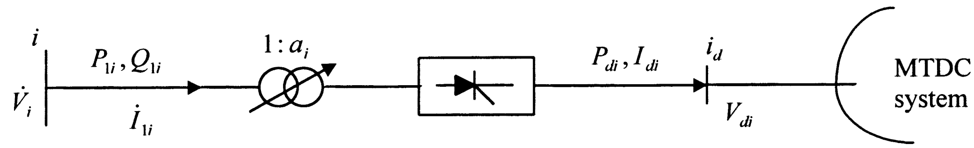

Fig. 1. Representation of a converter station.

P_{li}, Q_{li}: active and reactive power transmitted from bus i
I_{li}: AC current drawn from bus i.
ϕ_i: phase angle difference between $\dot{V}_i$ and $\dot{I}_{li}$
a_i: converter transformer tap
θ_i: converter control angle, firing angle for rectifier and extinction angle of advance for inverter
X_{ci}: communication reactance
P_{di}, I_{di}: DC injected power and current into bus I_d, respectively
V_{di}: DC voltage at bus i_d
ξ_i: 1 for rectifier, -1 for inverter
k_1: $3\sqrt{2/\pi}$
k_2: $3/\pi$
k_3: $0.995 \cdot k_1$
G_d: DC network conductance matrix.

4.3. Formulation

Compared with traditional AC state estimation, measurment equations which describe the injected active and reactive power must be modified to include the MTDC system. For those AC buses connected with converters, the equations for the real and reactive nodal powers can be expressed Eq. (25) and Eq. (26). Other AC measurements remain unchanged.

$$Z_{pi} = P_i^m = P_i(\text{ac}) + P_{di}(\text{dc}) = P_i\,(\text{ac}) + \xi_i V_i I_{1i} \cos\phi_i + \eta_{pi} \quad (25)$$

$$Z_{pi} = Q_i^m = Q_i(\text{ac}) + Q_{di}(\text{dc}) = Q_i\,(\text{ac}) + V_i I_{1i} \sin\phi_i + \eta_{qi} \quad (26)$$

From an inspection of a converter-connected AC bus model represented in Fig. 1, the equations of DC voltage, current and AC current drawn from AC bus to the converter may be written as the following four pseudo-measurements Z_{dc} and measurment $h_{\text{dc}}\,(X)$ per converter.

$$Z_{\text{dc}1} = 0 = V_{di} - k_1\, a_i\, V_i \cos\theta_i + \xi_i k_2 X_{ci} I_{di} + \eta_{d1} \quad (27)$$

$$Z_{\text{dc}2} = 0 = V_{di} - k_3\, a_i\, V_i \cos\phi_i + \eta_{d2} \quad (28)$$

$$Z_{\text{dc}3} = 0 = I_{1i} - k_1\, a_i\, I_{di} + \eta_{d3} \quad (29)$$

$$Z_{\text{dc}4} = 0 = I_{di} - \sum_{j=1}^{N_{\text{dc}}} G_{dij} V_{dj} + \eta_{d4} \quad (30)$$

In addition to the above pseudo-measurement equations, the actual measurement equations in DC system and AC/DC interface system are shown as follow:

$$Z_{\text{dc}5} = P_{li}^m = V_i I_{1j} \cos\phi_i + \eta_{d5} \quad (31)$$

$$Z_{\text{dc}6} = Q_{li}^m = V_i I_{1j} \sin\phi_i + \eta_{d6} \quad (32)$$

$$Z_{\text{dc}7} = I_{li}^m = I_{1j} + \eta_{d7} \quad (33)$$

$$Z_{\text{dc}8} = a_i^m = a_i + \eta_{d8} \quad (34)$$

$$Z_{\text{dc}9} = \cos\theta_i^m = \cos\theta_i + \eta_{d9} \quad (35)$$

$$Z_{\text{dc}10} = V_{di}^m = V_{di} + \eta_{d10} \quad (36)$$

$$Z_{\text{dc}11} = I_{di}^m = I_{di} + \eta_{d11} \quad (37)$$

$$Z_{\text{dc}12} = P_{di}^m = V_{di} I_{di} + \eta_{d12} \quad (38)$$

$$Z_{\text{dc}13} = P_{dij}^m = V_{di}\, G_{dij}(V_{dj} - V_{di}) + \eta_{d13} \quad (39)$$

$$Z_{\text{dc}14} = I_{di}^m = G_{dij}(V_{di} - V_{di}) + \eta_{d14} \quad (40)$$

Eqs. (25)–(40) form the set of the AC/MTDC system measurements Z. Among these equations, Eqs. (27)–(40) represent the MTDC system measurements Z_{dc}.

5. FACTS devices

In recent years, a number of advanced FACTS devices emerged with the rapid development of modern power electronics technology. This paper mainly deals with the three types: TCSC, TCPST and UPFC.

5.1. TCSC

Each TCSC introduces a state variable X_c into the state estimation. For the lines installed with TCSC devices, the line power flows P_{ij}, Q_{ij}, P_{ji}, Q_{ji} may be considered as the TCSC measurements (Fig. 2). So we can describe TCSC measurement equations as follows:

$$Z_{t\,\text{csc}\,1} = P_{ij}^m = V_i^2 G' \cos\theta_{ij} - V_i V_j B' \sin\theta_{ij} + \eta_{t\,\text{csc}\,1} \quad (41)$$

$$Z_{t\,\text{csc}\,2} = Q_{ij}^m = -V_i^2\left(B' + \frac{B_c}{2}\right) + V_i V_j B' \cos\theta_{ij} - V_i V_j G' \sin\theta_{ij} + \eta_{t\,\text{csc}\,2} \quad (42)$$

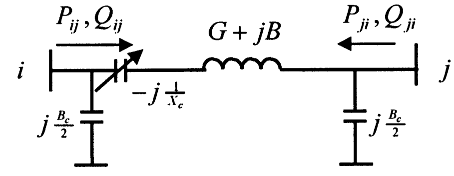

Fig. 2. Transmission line with a TCSC.

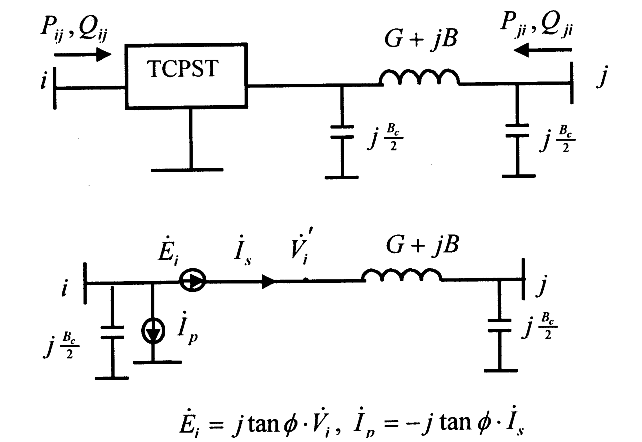

Fig. 3. (a) Transmission line with a TCPST; (b) schematic representation of a TCPST line.

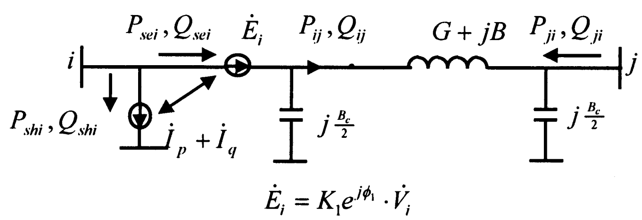

Fig. 4. Schematic representation of a UPFC line.

$$Z_{t\,\mathrm{csc}\,1} = P_{ij}^m = V_i^2 G' - V_i V_j G' \cos\theta_{ij} + V_i V_j B' \sin\theta_{ij} + \eta_{t\,\mathrm{csc}\,3} \quad (43)$$

$$Z_{t\,\mathrm{csc}\,4} = Q_{ij}^m = -V_i^2\left(B' + \frac{B_c}{2}\right) + V_i V_j B' \cos\theta_{ij} - V_i V_j G' \sin\theta_{ij} + \eta_{t\,\mathrm{csc}\,4} \quad (44)$$

where $G' = \dfrac{G^2+B^2}{G^2+[(G^2+B^2)X_c - B]^2}G$, $B' = \dfrac{G^2+B^2}{G^2+[(G^2+B^2)X_c - B]^2}[B-(G^2+B^2)X_c]$.

For the buses connected with TCSC lines, the measurment equations for the nodal powers should be modified as the following equations. Other AC measurment equations remain unchanged.

$$Z_{pi} = P_i^m = P_i(\mathrm{ac}) + \Delta P_{t\,\mathrm{csc}\,i} = P_i\,(\mathrm{ac}) - V_i^2 G'' + V_i V_j G'' \cos\theta_{ij} + V_i V_j B'' \sin\theta_{ij} + \eta_{pi} \quad (45)$$

$$Z_{qi} = Q_i^m = Q_i\,(ac) + \Delta Q_{t\,\mathrm{csc}\,i} = Q_i(\mathrm{ac}) - V_i^2 B'' + V_i V_j G'' \sin\theta_{ij} - V_i V_j B'' \cos\theta_{ij} + \eta_{qi} \quad (46)$$

$$Z_{pj} = P_i^m = P_j\,(\mathrm{ac}) + \Delta P_{t\,\mathrm{csc}\,j} = P_j\,(\mathrm{ac}) - V_j^2 G'' + V_i V_j G'' \cos\theta_{ij} - V_i V_j B'' \sin\theta_{ij} + \eta_{pj} \quad (47)$$

$$Z_{qj} = Q_i^m = Q_j(\mathrm{ac}) + \Delta Q_{t\,\mathrm{csc}\,j} = Q_j(\mathrm{ac}) + V_j^2 B'' - V_i V_j G'' \sin\theta_{ij} - V_i V_j B'' \cos\theta_{ij} + \eta_{qi} \quad (48)$$

where $G'' = G' - G$, $B'' - B$.

5.2. TCPST

According to the equivalent model of TCPST [7], we know a new state variable φ is appended. For the lines installed with TCPST devices, the line power flows, P_{ij}, Q_{ij}, P_{ji}, Qji may be considered as the TCPST measurements (Fig. 3). Then we can describe TCPST measurment equations as follow:

$$Z_{\mathrm{tcpst}\,1} = P_{ij}^m = V_i^2 G/K^2 - V_i V_j(G\cos(\theta_{ij}+\varphi) + B\sin(\theta_{ij}+\varphi))/K + \eta_{\mathrm{tcpst}\,1} \quad (49)$$

$$Z_{\mathrm{tcpst}\,2} = Q_{ij}^m = -V_i^2\left(B\Big/K^2 + \frac{B_c}{2}\right) + V_i V_j(B\cos(\theta_{ij}+\varphi) - G\sin(\theta_{ij}+\varphi))/K + \eta_{\mathrm{tcpst}\,2} \quad (50)$$

$$Z_{\mathrm{tcpst}\,3} = P_{ji}^m = V_j^2 G - V_i V_j(G\cos(\theta_{ij}+\varphi) - B\sin(\theta_{ij}+\varphi))/K + \eta_{\mathrm{tcpst}\,3} \quad (51)$$

$$Z_{\mathrm{tcpst}\,4} = Q_{ji}^m = -V_j^2\left(B + \frac{B_c}{2}\right) + V_i V_j(B\cos(\theta_{ij}+\varphi) + G\sin(\theta_{ij}+\varphi))/K + \eta_{\mathrm{tcpst}\,4} \quad (52)$$

where $K = \cos\varphi$.

The measurment equations for the nodal injected powers of the TCPST buses should be modified as the following equations. Other AC measurement equations remain unchanged.

$$Z_{pi} = P_i^m = P_i(\mathrm{ac}) + \Delta P_{\mathrm{tcpst}i} = P_i(\mathrm{ac}) - V_i^2 T^2 G - V_i V_j T\,(G\sin\theta_{ij} - B\cos\theta_{ij}) + \eta_{pi} \quad (53)$$

$$Z_{qi} = Q_i^m = Q_i(\mathrm{ac}) + \Delta Q_{\mathrm{tcpst}i} = Q_i\,(\mathrm{ac}) + V_i^2 T^2 B + V_i V_j T\,(G\cos\theta_{ij} + B\sin\theta_{ij}) + \eta_{qi} \quad (54)$$

$$Z_{pj} = P_j^m = P_j(\mathrm{ac}) + \Delta P_{\mathrm{tcpst}j} = P_j\,(\mathrm{ac}) - V_i V_j T\,(G\sin\theta_{ij} + B\cos\theta_{ij}) + \eta_{pj} \quad (55)$$

$$Z_{qj} = Q_j^m = Q_j(\mathrm{ac}) + \Delta Q_{\mathrm{tcpst}j} = Q_j\,(\mathrm{ac}) - V_i V_j T\,(G\cos\theta_{ij} - B\sin\theta_{ij}) + \eta_{qi} \quad (56)$$

where $T = \tan\varphi$.

5.3. UPFC

UPFC, a versatile FACTS device, has the unique capability to control simultaneously both the voltage magnitude and active and reactive power flows on a transmission corridor. Fig. 4 shows the equivalent model of the UPFC line. Three indepdent variables $X_{\mathrm{upfc}} = [K_1,\ \phi_1,\ I_q]^T$ are introduced because of each installed UPFC [7].

After regarding the power flows $P_{\mathrm{se}i}$, $Q_{\mathrm{se}i}$ and $P_{\mathrm{sh}i}$, $Q_{\mathrm{sh}i}$ in the series and shunt parts of the UPFC, and the line power flows P_{ij}, Q_{ij} and P_{ji}, Q_{ji} as the UPFC measurements, we can write the UPFC measurement equations as:

$$Z_{\mathrm{upfc}\,1} = P_{\mathrm{se}i}^m = (G + K_1\cos\phi_1)\,V_i^2 - K_1\sin\phi_1 \left(B + \frac{B_c}{2}\right) V_i^2 - V_i V_j(G\cos\theta_{ij} + B\sin\theta_{ij}) + \eta_{\mathrm{upfc}\,1} \quad (57)$$

$$Z_{\mathrm{upfc}\,2} = Q_{\mathrm{se}i}^m = -\left(B + \frac{B_c}{2}\right)(1 + K_1\cos\phi_1)V_i^2 - K_i V_i^2 G\sin\phi_i - V_i V_j(G\sin\theta_{ij} - B\cos\theta_{ij}) + \eta_{\mathrm{upfc}\,2} \quad (58)$$

$$Z_{\mathrm{upfc}\,3}=P_{\mathrm{sh}i}^{m}=K_1^2V_i^2G+K_1V_i^2\left[G\cos\phi_1+\left(B+\frac{B_c}{2}\right)\sin\phi_1\right]-K_1V_iV_j[G\cos(\theta_{ij}+\phi_1)+B\sin(\theta_{ij}+\phi_1)]+\eta_{\mathrm{upfc}\,3} \quad (59)$$

$$Z_{\mathrm{upfc}\,4}=Q_{\mathrm{sh}i}^{m}=V_iI_q+\eta_{\mathrm{upfc}\,4} \quad (60)$$

$$Z_{\mathrm{upfc}\,5}=P_{ij}^{m}=(1+K_1^2+2K_1\cos\phi_1)\,V_i^2G-V_iV_j(G\cos\theta_{ij}+B\sin\theta_{ij})-K_1V_iV_j[G\cos(\theta_{ij}+\phi_1)+B\sin(\theta_{ij}+\phi_1)]+\eta_{\mathrm{upfc}\,5} \quad (61)$$

$$Z_{\mathrm{upfc}\,6}=Q_{ij}^{m}=-(1+K_1^2+2K_1\cos\phi_1)\,V_i^2\left(B+\frac{B_c}{2}\right)+V_iV_j(B\cos\theta_{ij}-G\sin\theta_{ij})+K_1V_iV_j[B\cos(\theta_{ij}+\phi_1)-G\sin(\theta_{ij}+\phi_1)]+\eta_{\mathrm{upfc}\,6} \quad (62)$$

$$Z_{\mathrm{upfc}\,7}=P_{ji}^{m}=V_i^2G-V_iV_j\,(G\cos\theta_{ij}-B\sin\theta_{ij})-K_1V_iV_j[G\cos(\theta_{ij}+\phi_1)-B\sin(\theta_{ij}+\phi_1)]+\eta_{\mathrm{upfc}\,7} \quad (63)$$

$$Z_{\mathrm{upfc}\,8}=Q_{ji}^{m}=-V_i^2\left(B+\frac{B_c}{2}\right)+V_iV_j(B\cos\theta_{ij}+G\sin\theta_{ij})+K_1V_iV_j\,[B\cos(\theta_{ij}+\phi_1)+G\sin(\theta_{ij}+\phi_1)]+\eta_{\mathrm{upfc}\,8} \quad (64)$$

The measurement equations for the nodal injected powers of the UPFC buses should be modified as the following equations. Other AC measurment equations remain unchanged.

$$Z_{pi}=P_i^m=P_i(\mathrm{ac})+\Delta P_{\mathrm{upfc}\,i}=P_i\,(\mathrm{ac})-G\,[2K_1V_i^2\cos\phi_1+K_1^2V_i^2-K_1V_iV_j\cos(\phi_1+\theta_{ij})]+BK_1V_iV_j\sin(\phi_1+\theta_{ij})+\eta_{pi} \quad (65)$$

$$Z_{pi}=Q_i^m=Q_i\,(\mathrm{ac})+\Delta Q_{\mathrm{upfc}\,i}=Q_i(\mathrm{ac})+K_1V_i^2G\sin\phi_1+K_1V_i^2\,B\cos\phi_1+\frac{B_c}{2}K_1V_i^2\cos\phi_1-V_iI_q+\eta_{qi} \quad (66)$$

$$Z_{pj}=P_j^m=P_j\,(\mathrm{ac})+\Delta P_{\mathrm{upfc}\,j}=P_j\,(\mathrm{ac})+K_1V_iV_j\,[G\cos(\theta_{ij}+\phi_1)-B\sin(\theta_{ij}+\phi_1)]+\eta_{pj} \quad (67)$$

$$Z_{qi}=Q_j^m=Q_j\,(\mathrm{ac})+\Delta Q_{\mathrm{upfc}\,j}=Q_j\,(\mathrm{ac})-KV_iV_j[B\cos(\theta_{ij}+\phi_1)+G\sin(\theta_{ij}+\phi_1)]\eta_{qi} \quad (68)$$

6. Test cases

A state estimation program, based on the proposed improved sequential method, has been developed to include the FACTS devices and MTDC systems. The developed program has been extensively tested on a number of power systems embedded with FACTS devics and DC links. The solution accuracy, convergence behavior and computational efficiency of the proposed method are verified by the test results and compared with those obtained from WLS and FDSE estimators. Three different networks are presented as follow.

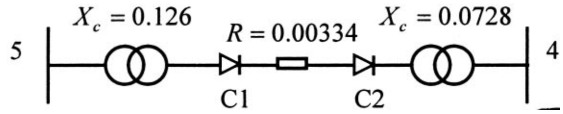

Fig. 5. A DC link in IEEE 14-bus system (values in p.u.).

1. IEEE 14-bus system with a DC link and a TCSC.
2. IEEE 14-bus system with a DC link and a TCSC.
3. IEEE 30-bus system with a two TCSCs, a TCPST and a UPFC.

Appropriate weighting factors are calculated from the following considerations.

1. For AC system and FACTS devices
   $\sigma=0.002|\text{meter readings}|+0.0035|\text{full scale}|$
2. For MTDC system
   $\sigma=0.001|\text{meter readings}|+0.002|\text{full scale}|$.

The full scale values for all meters are assumed 1.0 p.u. The data for testing the improved sequential state estimation are simulatd using the results from power flow analysis. The performance of the algorithm is assessed by comparing the estimated values $\hat{Z}$ and the measured values Z with the true value Z' given by the power flow results. The followign indices are used for comparison [3]:

$$J_M=\frac{1}{M}\sum_{i=1}^{m}[(Z_i-Z_i^t)/\sigma_i]^2 \quad (69)$$

$$J_E=\frac{1}{m}\sum_{i=1}^{m}[(\hat{Z}_i-Z_i^t)/\sigma_i]^2 \quad (70)$$

$$R_{\mathrm{av}}=\frac{1}{m}\sum_{i=1}^{m}(|\hat{Z}_i-Z_i^t|/\sigma_i) \quad (71)$$

$$R_{\max}=\max_{i=1}^{m}(|\hat{Z}_i-Z_i^t|/\sigma_i) \quad (72)$$

The performace index J_M shows the level of uncertainty in the measurements. The index J_E indicates how close the estimated values are to the true ones. The effectivenss of the filtering process is indicated by a value of J_E less than the corresponding value of J_M. The indices R_{av} and $R_{\max}$ show the average and maximum value of weighted residuals from the true values.

6.1. Test system 1

The IEEE 14-bus system is modified to include a DC link connected between nodes 4 and 5, as shown in Fig. 5, and a TCSC installed in the line 2–3. The original line between nodes 4 and 5 is removed.

Three different methods, the basic WLS, FDSE and ISQ (Improved Sequential method) are applied to solve the system. Tables 1a, 1b and 1c show the state estimation results of these three different state estimators. Apparently an inspection of the results indicates that the ISQ method can converge to acceptable state with the required tolerance of 10^{-5} p.u. From Tables 1a, 1b

Table 1a
State estimation results of 14-bus AC/MTDC system (test system 1) — AC state results (p.u.)

Bus no.	True values		WLS		ISQ		FDSE	
	V^t	θ^t (°)	$\hat{V}$	$\hat{\theta}$ (°)	$\hat{V}$	$\hat{\theta}$ (∘)	$\hat{V}$	$\hat{\theta}$ (°)
1	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000
2	1.0450	−5.0435	1.0448	−5.0441	1.0448	−5.0442	1.0449	−5.0446
3	1.01000	−11.6931	1.0092	−1.6986	1.0093	−11.6998	1.0101	−11.6991
4	1.0149	−9.9260	1.0142	−9.9292	1.0141	−9.9298	1.0147	−9.9311
5	0.9982	−8.4096	0.9966	−8.4279	0.9971	−8.4192	0.9973	−8.4028
6	1.0700	−13.9566	1.0699	−13.9844	1.0701	−13.9850	1.0701	−13.9761
7	1.0603	−12.9950	1.0604	−13.0274	1.0604	−13.0274	1.0498	−13.0222
8	1.0900	−12.9950	1.0901	−13.0274	1.0901	−13.0274	1.0901	−13.0222
9	1.0548	−13.5827	1.0901	−13.0274	1.0901	−13.0274	1.0901	−13.0222
10	1.05051	−14.7573	1.0502	−14.7891	1.0500	−14.7753	1.0488	−14.7752
11	1.0565	−14.4872	1.0565	−14.5171	1.0564	−14.5092	1.0564	−14.5015
12	1.0568	−14.5420	1.0567	−14.5692	1.0567	−14.5698	1.0569	−14.5618
13	1.0503	−14.7980	1.0502	−14.8251	1.0503	−14.8169	1.0502	−14.8321
14	1.0349	−15.6778	1.0346	−15.6980	1.0346	−15.6931	1.0347	−15.7010
Iteration number			5		5		7	

Table 1b
State estimation results of 14-bus AC/MTDC system (test system 1) — DC state results (p.u.)

		V_d	I_d	a	θ(°)	I_1	ϕ (°)
C1	X^t	1.2807	0.4576	1.0006	8.0000	0.6182	17.4034
	WLS	1.2812	0.4574	1.0004	8.0110	0.6155	17.4112
	ISQ	1.2812	0.4573	1.0004	8.0098	0.6153	17.4177
	FDSE	1.2810	0.4567	1.0005	8.0102	0.6147	17.4094
C2	X^t	1.2791	−0.4576	0.9950	16.000	−0.6147	−19.4986
	WLS	1.2796	−0.4575	0.9951	15.9762	−0.6136	−19.4987
	ISQ	1.2796	−0.4575	0.9949	15.9843	−0.6136	−19.4963
	FDSE	1.2794	−0.4569	0.9949	15.9644	−0.6143	−19.4976

and 1c, it is noted that different iteration number for each method is required to converge. But clearly the proposed ISQ method, approximately requiring the same number as the WLS method, is superior to that of the FDSE approach.

6.2. Test system 2

In the IEEE 14-bus system, AC lines 2–4 and 4–5 are removed and a 3-terminal DC mesh system is included between nodes 2, 4 and 5. Additionally a WSC is installed in the line 1–5 and a TCPST is in the line 6–13 to form the test system 2. Detailed DC system information can be found in [8]. Tables 2a, 2b and 2c reports the system state estimation results and shows that the proposed ISQ algorithm is successful.

6.3. Test system 3

The IEEE 30-bus system with two TCSCs in the line 2–4 and line 6–28, respectively, a TCPST in the line 12–15 and a UP17C in the line 10–22, is tested as test system 3. To solve the above test system, the WLS requires five iterations to reach convergence with the required tolerance. The proposed ISQ method converges after six iterations. However, it costs the FDSE method ten iteration times to reach convergence. Table 3 presents the state estimation FACTS results of the test system 3 and also indicates that the proposed approach performs satisfactorily.

The performance indices and a quantitative assessment of the accuracy of the proposed estimation method are given in Table 4 for the above three test

Table 1c
State estimation results of 14-bus AC/MTDC system (test system 1) — TCSC state results (p.u.)[a]

	X^t	WLS	ISQ	FDSE
X_c	−0.04300	−0.04305	−0.04303	−0.04303

[a] X^t stands for the true value.

Table 2a
State estimation results of 14-bus AC/MTDC system (test system 2) — AC state results (p.u.)

Bus no.	True values		WLS		ISQ		FDSE	
	V^t	θ^t (°)	$\hat{V}$	$\hat{\theta}$ (°)	$\hat{V}$	$\hat{\theta}$ (°)	$\hat{V}$	$\hat{\theta}$ (°)
1	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000
2	1.0450	−4.7999	1.0448	−4.8010	1.0451	−4.8015	1.0450	−4.8010
3	1.0100	−12.3419	1.0088	−12.3533	1.0102	−12.3605	1.0103	−12.3600
4	0.9873	−8.9514	0.9867	−8.9545	0.9870	−8.9508	0.9871	−8.9513
5	0.9969	−7.9162	0.9963	−7.9264	0.9966	−7.9203	0.9965	−7.9208
6	1.0700	−13.5653	1.0700	−13.5794	1.0702	−13.6192	1.0701	−13.6187
7	1.0478	−12.1015	1.0478	−12.1154	1.0477	−12.1158	1.0476	−12.1153
8	1.0900	−12.1015	1.0997	−12.1154	1.0901	−12.1158	1.0903	−12.1153
9	1.0429	−13.7048	1.0430	−13.7189	1.0433	−13.7112	1.0430	−13.7107
10	1.0403	−13.9635	1.0404	−13.9777	1.0403	−14.0019	1.0404	−14.0014
11	1.0515	−13.8859	1.0515	−13.9000	1.0516	−13.8893	1.0516	−13.8887
12	1.0559	13.8649	1.0560	−13.8790	1.0559	−13.8858	1.0560	−13.8753
13	1.0494	−13.6770	1.0494	−13.6911	1.0491	−13.7040	1.0491	−13.6995
14	1.0276	−14.7057	1.02741	−14.7199	1.0275	−14.7118	1.0276	−14.7113
Iteration number			5		6		11	

Table 2b
State estimation results of 14-bus AC/MTDC system (test system 2) — DC state results (p.u.)

		V_d	I_d	a	θ (°)	I_1	ϕ (°)
C1	X^t	1.2860	0.4805	1.0134	12.5	0.6770	18.6655
	WLS	1.2860	0.4804	1.0167	12.5013	0.6743	18.6670
	ISQ	1.2860	0.4820	1.0151	12.5102	0.6743	18.6683
	FDSE	1.2862	0.4820	1.0152	12.5114	0.6741	18.6679
	X^t	1.2856	0.4355	0.9788	17.8419	0.5757	20.6875
	WLS	1.2858	0.4351	0.9789	17.8290	0.5747	20.6831
	ISQ	1.2858	0.4351	0.9787	17.8458	0.5745	20.6838
	FDSE	1.2860	0.4352	0.9787	17.8226	0.5746	20.6835
C3	X^t	1.2795	−0.9160	1.0464	19.5321	−1.2944	−22.7815
	WLS	1.2795	−0.9155	1.0474	19.4634	−1.2933	−22.7468
	ISQ	1.2795	−0.9154	1.0461	19.4927	−1.2919	−22.7471
	FDSE	1.2797	−0.9160	11.0455	119.4795	−1.2910	−22.7465

Table 2c
State estimation results of 14-bus AC/MTDC system (test system 2) — TCSC and TCPST state results (p.u.)

Device	Variable	X^t	WLS	ISQ	FDSE
TCSC	X_c	−0.02935	−0.02932	−0.02927	−0.02928
TCPST	φ (°)	0.98056	0.98057	0.98056	0.98056

Table 3
FACTS state results for test system 3 (p.u.)

FACTS device	Variable	X^t	WLS	ISQ	FDSE
TCSC (2–4)	X_c	−0.05414	−0.05449	−0.05455	−0.05451
TCSC (6–28)	X_c	−0.03599	−0.03614	−0.03619	−0.03612
TCPST (12–15)	φ (°)	0.63822	0.63822	0.63825	0.63825
UPFC (10–22)	K_1	0.00573	0.00570	0.00569	0.00569
	ϕ_1 (°)	52.4629	52.3661	52.3574	52.3602
	I_q	−0.02583	−0.02606	−0.02613	−0.02610

Table 4
Performance indices

	J_M	J_E	R_{av}	R_{max}
Test system 1	0.5762	0.2873	0.2718	0.8098
Test system 2	0.1109	1.0.0741	0.1302	0.9252
Test system 3	0.3619	1.0.1126	0.3973	1.2579

systems. The maximum error is acceptable and the filtering process of the proposed algorithm is successful since the ratio J_E/J_M is always less than 1.

7. Conclusion

This paper has presented a novel and efficient approach called improved sequential method, suitable for the state estimation of power systems embedded with FACTS devices and MTDC systems. By the decoupling of AC, FACTS and MTDC parts in the gain matrix through a mathematical process, the algorithm retains good convergence property as the conventional WLS method and its robustness is verified by extensive tests on several systems. The proposed method is based on a sequential iterative form and the polar coordinates formulation of the AC, MTDC and FACTS equations. Moreover, it possesses its main merit of easily extending existing AC state estimators to include the effects of WDC systems and FACTS devices.

8. Appendix

$$\Delta\tilde{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{(k)},X_{F}^{(k)}) = \Delta\bar{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{(k)},X_{F}^{(k)})$$
$$- \bar{H}_{ac-dc}^{(k)}\bar{H}_{dc}^{+\,(k)},\Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)})$$
$$- \bar{H}_{ac-F}^{(k)}\bar{H}_{F}^{+\,(k)}\Delta\bar{Z}_{F}(X_{ac}^{(k)},X_{F}^{(k)})$$
$$= \Delta\bar{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{(k)},X_{dc}^{(k)},X_{F}^{(k)})$$
$$- \bar{H}_{ac-dc}^{(k)}\Delta X_{dc}^{\mathrm{temp}\,(k)} - \bar{H}_{ac-F}^{(k)}\Delta X_{F}^{\mathrm{temp}\,(k)} = R^{-1/2}$$
$$[Z - h(X_{ac}^{(k)},X_{dc}^{(k)},X_{F}^{(k)}) - H_{ac-dc}^{(k)}\Delta X_{dc}^{\mathrm{temp}\,(k)}$$
$$- H_{ac-F}^{(k)}\Delta X_{F}^{\mathrm{temp}\,(k)}] = \Delta\bar{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{(k)} + \Delta X_{dc}^{\mathrm{temp}\,(k)},X_{F}^{(k)}$$
$$+ \Delta X_{F}^{\mathrm{temp}\,(k)}) = \Delta\bar{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{\mathrm{temp}\,(k+1)},X_{F}^{\mathrm{temp}\,(k+1)})$$
$$= \Delta\bar{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{(k)} + \Delta X_{dc}^{temp\,(k)},X_{F}^{(k)} + \Delta X_{F}^{\mathrm{temp}\,(k)})$$
$$= \Delta\bar{Z}_{ac}(X_{ac}^{(k)},X_{dc}^{\mathrm{temp}\,(k+1)},X_{F}^{\mathrm{temp}\,(k+1)}) \tag{A-1}$$

Here a detailed derivation of Eq. (19) is presented. From Eq. (9) we have

$$\bar{H}_{dc-ac}\Delta X_{ac}^{(k)} + \bar{H}_{dc}\Delta X_{dc}^{(k)} \cong \Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)}) \tag{A-2}$$

Because

$$\Delta X_{dc}^{(k)} = \Delta X_{dc}^{\mathrm{temp}\,(k)} + \Delta X_{dc}^{\mathrm{com}\,(k)} \tag{A-3}$$

Then

$$\bar{H}_{dc-ac}\Delta X_{ac}^{(k)} + \bar{H}_{dc}(\Delta X_{dc}^{\mathrm{temp}\,(k)} + \Delta X_{dc}^{\mathrm{com}\,(k)})$$
$$\cong \Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)}) \tag{A-4}$$

And because

$$\Delta X_{dc}^{\mathrm{temp}\,(k)} = \bar{H}_{dc}^{+}\Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)}) \tag{A-5}$$

From Eq. (A-4) and Eq. (A-5) we can get

$$\bar{H}_{dc-ac}\Delta X_{ac}^{(k)} + \bar{H}_{dc}\Delta X_{dc}^{\mathrm{com}\,(k)} \cong [I - \bar{H}_{dc}$$
$$\bar{H}_{dc}^{+}]\Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)}) \tag{A-6}$$

where I is the identity matrix. Pre-multiplying (A-6) by the pseudo-inverse matrix. Pre-multiplying (A-6) by the pseudo-inverse matrix $\bar{H}_{dc}^{+}$ leads to

$$\bar{H}_{dc}^{+}(\bar{H}_{dc-ac}\Delta X_{ac}^{(k)} + \bar{H}_{dc}\Delta X_{dc}^{\mathrm{com}\,(k)}) \cong \bar{H}_{dc}^{+}[I - \bar{H}_{dc}$$
$$\bar{H}_{dc}^{+}]\Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)}) = [\bar{H}_{dc}^{T}\bar{H}_{dc}]^{-1}\bar{H}_{dc}^{T}$$
$$(1 - \bar{H}_{dc}[\bar{H}_{dc}^{T}\bar{H}_{dc}]^{-1}\bar{H}_{dc}^{T})\Delta\bar{Z}_{dc}(X_{ac}^{(k)},X_{dc}^{(k)}) = 0 \tag{A-7}$$

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References

[1] F.F. Wu, Power system state estimation: a survey, Elec. Power Energy Syst. 12 (2) (1990) 80–87.
[2] M.B. Do Coutto Filho, A.M. Leite da Silva, D.M. Falcao, Bibliography on power system state estimation (1968–1989), IEEE Trans. Power Syst. 5 (3) (1990) 950–961.
[3] R. Jegatheesan, K. Duraiswamy, AC/Multi-terminal DC power system state estimation — a sequential approach, Elec. Machines and Power Syst. 12 (1987) 27–42.
[4] L. Roy, A.K. Sinha, H.N.P. Srivastava, Static state estimation of multitenninal DC/AC power system in rectangular coordinates, Elec. Mach. Power Syst. 19 (1991) 69–80.
[5] A.K. Sinha, L. Roy, H.N. Srivastava, Adecoupled second order state estimator for AC-DC power systems, IEEE Trans. Power Syst. 9 (3) (1994) 1485–1491.
[6] A. Monticelli, A. Garcia, Fast decoupled state estimators, IEEE Trans. Power Syst. 5 (2) (1990) 556–564.
[7] G. Shaoyun, Optimal Power System Operation and Control Incorporating FACTS Devices, PhD Dissertation, The HongKong Polytechnic University, August 1998.
[8] D. Qifeng, Z. Boming, A new approach to AC/MTDC power flow, International Conference On Advances of Power System Control, Operation and Management, Hong Kong, 1997, pp. 689–694.